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UNCERTAINTY IN ECONOMICAL ANALYSIS OF SOLAR WATER HEATING AND PHOTOVOLTAIC SYSTEMS

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Abstract—This paper focuses on the statistical analysis of the energy fraction supplied by solar energy for solar water heating systems based on the *f*-chart method. An analysis is also presented for photovoltaic systems, where costs are linearly proportional to collector area. The uncertainty of the solar fraction is correlated with the monthly means of the global irradiation and the correlation coefficient between monthly means. Numerical examples for one location in Brazil and three locations in the United States are presented. These examples show that the uncertainty of the life cycle savings is significantly dependent on the uncertainty of the monthly means of the solar radiation data. The present analysis intends to provide a basic procedure that could be useful to make a straightforward feasibility analysis of a solar system. This is particularly interesting to evaluate the investment risk associated with photovoltaic plants, for which the capital costs are comparable to the advantages in saving electric energy from the utility grid under present scenarios in most places.

1. INTRODUCTION

The production of PV modules has been increasing in recent years, while production costs have been decreasing and are set to decrease further due to new manufacturing technologies and production scale factors. Today, PV modules are sold at about US$ 3–5/Wp, and for both the US Department of Energy (Zweibel, 1999) and the European Commission (Bruton *et al*., 1996) production cost goal for thin films are under US$ 0.50/Wp, which is visualized with current technology, that will further evolve and could lead to even lower production costs. Under these assumptions, PV module market prices under US$ 1.00/Wp are possible, with PV systems priced at about twice this figure. On the other hand, the cost of energy derived from fossil fuel sources and hydroelectricity are pressed to go up, due to the increasing penalty for environment degradation and pollution, the requirements for increased investments in exploration and to the decrease of the availability of fossil fuel reserves. The search for new alternatives to produce pollution-free energy is nowadays included in government planning worldwide. Solar energy, in this context, can be considered a true competitive alternative for the near future.

Prior to making a decision on any alternative energy project, one should look for an economical figure of merit. The methods for economical analysis presently in use in solar energy projects include the Life Cycle Cost (LCC), Life Cycle Savings (LCS), Annualized Life Cost (ALC), Payback Time and Return of Investment (ROI). These are described in Duffie and Beckman (1991), and many standard books on economics. Among these, a useful and straightforward technique for LCS to optimize solar heating and cooling systems is the $P_1 - P_2$ method proposed in Brandemuehl and Beckman (1979).

Sensibility analysis is useful in order to evaluate the effect of design parameter variation, as well as the effect of inflation, interest rates, fuel cost variation, and capital cost on the LCS. A complete analysis is also carried out in Brandemuehl and Beckman (1979). In the circumstance where the capital cost of any alternative energy plant becomes close to the threshold cost, the precise knowledge of the availability of primary energy resources becomes of major importance. In the case of PV generation therefore, the availability of solar radiation data, its variability as well as the uncertainty of the monthly

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means of the global radiation, should be taken into account to evaluate the uncertainty of LCS.

This paper presents, for both solar water heating and photovoltaics, an elementary uncertainty analysis of LCS, as a function of the monthly means of global radiation through the monthly means of the total radiation incident on the tilted surface of the thermal collector or PV modules. The analysis will be carried out for water heating systems, and also for PV systems integrated to the utility grid, as reported in Rüther (1998, 2000), and Rüther and Dacoregio (2000). Data from a 2 kWp grid-connected, building-integrated, thin film amorphous silicon (a-Si) PV system operating in Brazil since 1997, which is fully and continuously monitored, are used in the analysis. The latitude-tilted PV system operates in Florianopolis (27° S), under an annual global horizontal irradiation level of over 1600 kWh/m^2, which is one of the lowest irradiation levels in the country (Colle *et al*., 1999a). This site can thus be regarded as a worst-case scenario for operating PV in Brazil; nevertheless an annual energy yield of 1300 kWh/kWp has been reached, with an AC performance ratio of 85% (Rüther, 2000). The analysis for PV in this paper is based on the above-mentioned a-Si PV system. For a-Si, due to the opposite and matching effects of thermal annealing of the Staebler Wronski effect (Staebler and Wronski, 1977), which improves performance at higher operating temperatures, and the small temperature coefficient of power, which hinders performance at higher operating temperatures, temperature effects on performance can be neglected. The same holds for thin film CdTe, which has also a small temperature coefficient of power, but for the traditional bulk crystalline silicon technologies (both single and polycrystalline), and for the new CIS-related (copper indium disselenide) PV technologies, temperature effects have to be taken into account. For these PV technologies, high operating temperatures can significantly reduce performance, an especially important fact to consider for PV systems operating in warm climates like Brazil (Rüther, 1996, 2000).

Large uncertainties may arise from modeling correlations for the radiation incident on tilted surfaces. In spite of the fact that these uncertainties can be significant, the numerical analysis will take into account only the uncertainties arising from the monthly means of global radiation on the horizontal surface.

The monthly means of global radiation are usually estimated from data of sunshine duration records or from pyranometer data collected in ground stations of meteorological services. The former are available in many countries for long-term periods, up to 30 years. The monthly means derived from sunshine duration records are less accurate than those measured by calibrated pyranometers. On the other hand, time series longer than 30 years of qualified data from pyranometers are seldom available, particularly for South American countries, as reported in Tsvetkov (1997).

The assessment of solar radiation from satellites has become a useful way to derive monthly means of incoming global radiation on a horizontal surface. Bias errors less than 5% and mean square errors around 7% for monthly means are usually found by many authors (Zelenka *et al*., 1992; Stuhlmann *et al*., 1990; Pereira *et al*., 1996; Pinker and Lazlo, 1992). Presently, data derived from satellites are seldom available for periods longer than 10 years. The sampling of monthly means derived from satellites is therefore limited and statistically not representative. On the other hand, for many countries, satellite-derived data are the only possibility to assess solar energy, as is the case of Brazil (Colle *et al*., 1999b) and many South American countries. Therefore, before going to study the economical impact of solar energy in the energy market, one should take into account the effect of the uncertainty of the solar radiation data on the economical analysis.

2. UNCERTAINTY ANALYSIS

As mentioned before, two systems will be analyzed in order to cover solar domestic hot water systems and PV applications as follows.

2.1. Case A: solar domestic hot water system (SDHWS)

According to Duffie and Beckman (1991), for the $P_1 - P_2$ method LCS is given by the following equation

$$\mathrm{LCS} = P_1 C_{F1} FL - P_2(C_A A_c + C_E) \tag{1}$$

where A_c is the collector area, C_{F1} is the cost of the auxiliary energy (US$/GJ) in the first year of the economical analysis, F is the annual solar fraction, L is the annual load (GJ), C_A is the collector cost per square metre (US$/m^2), C_E is the cost of the system independent of the collector area, and P_1 and P_2 are economical factors, accounting for the present value of the series of yearly savings, mortgage payments, insurance cost, depreciation and other costs.

The annual fraction F is defined as

$$F = \sum_{i=1}^{12} f_i L_i / L \tag{2}$$

where f_i is the monthly fraction of solar energy and L_i is the monthly load (GJ). For solar collectors with liquid fluids, f_i is a function of parameters X_i and Y_i as follows

$$f_i = 1.029Y_i - 0.065X_i - 0.245Y_i^2 + 0.0018X_i^2 + 0.0215Y_i^3 \tag{3}$$

where X_i is the monthly ratio of the heat loss of the solar collector to the monthly load given by

$$X_i = (F_R U_L)(F'_R/F_R)(T_{\text{ref}} - \overline{T}_a)\Delta t \frac{A_c}{L_i} \tag{4}$$

and Y_i is the monthly ratio of the solar energy absorbed by the collector to the monthly load given by

$$Y_i = F_R(\tau\alpha)_n(F'_R/F_R)(\overline{\tau\alpha})/(\tau\alpha)_n \overline{H}_{Ti} N_i A_c / L_i. \tag{5}$$

The monthly mean of the solar radiation incident on a tilted surface, $\overline{H}_{Ti}$, is related to the monthly mean of the global and the diffuse radiation on a horizontal surface. There are several correlations derived from the hourly sums method (HSM), daily sums method (DSM), as reported in Behr (1997), Reindl *et al.* (1990), and Hay and McKay (1985). These correlations however are not available analytically in terms of the monthly mean. Therefore the correlation for monthly means given by Hay (1979) will be adopted here. This correlation can be written in non-dimensional form as a function of the monthly average of the daily clearness index $\overline{K}_T$ as given in Appendix B. In this case Y_i can take the form

$$Y_i = F_R(\tau\alpha)_n(F'_R/F_R)\overline{H}_{0i} N_i A_c \psi(\overline{K}_{Ti})/L_i \tag{6}$$

where $\overline{H}_{0i}$ is the monthly mean of daily sums of solar radiation on the top of the atmosphere and $\psi(\overline{K}_{Ti})$is given by,

$$\psi(\overline{K}_{Ti}) = (\overline{\tau\alpha})/(\tau\alpha)_n \overline{H}_{Ti}/\overline{H}_{0i}. \tag{7}$$

The equation for the uncertainty of LCS as a function of $\overline{H}_i$ can be derived as follows

$$\delta \text{LCS} = \sum_{i=1}^{12} (\partial \text{LCS}/\partial \overline{H}_i)\delta \overline{H}_i. \tag{8}$$

The partial derivative of Eq. (1) leads to

$$\partial \text{LCS}/\partial \overline{H}_i = P_1 C_{F1} \partial(FL)/\partial \overline{H}_i. \tag{9}$$

From Eq. (2) results

$$\partial(FL)/\partial \overline{H}_i = L_i \partial f_i/\partial \overline{H}_i. \tag{10}$$

By using the chain rule, the partial derivative of Eq. (3) leads to

$$\partial f_i/\partial \overline{H}_i = \partial f_i/\partial X_i \, \partial X_i/\partial \overline{H}_i + \partial f_i/\partial Y_i \, \partial Y_i/\partial \overline{H}_i \tag{11}$$

where

$$\partial f_i/\partial Y_i = 1.029 - 0.49Y_i + 0.0645 f_i^2 \tag{12}$$

and $\partial X_i/\partial \overline{H}_i$ vanishes.

From Eqs. (9)–(11), Eq. (8) takes the form

$$\delta \text{LCS} = P_1 C_{F1} \sum_{i=1}^{12} g(Y_i) L_i \partial Y_i/\partial \overline{H}_i \tag{13}$$

where $g(Y_i) = \partial f_i/\partial Y_i$.

From Eq. (6),

$$\partial Y_i/\partial \overline{H}_i = F_R(\tau\alpha)_n(F'_R/F_R)\overline{H}_{0i} N_i A_c (\mathrm{d}\psi/\mathrm{d}\overline{K}_{Ti}) \times (\mathrm{d}\overline{K}_{Ti}/\mathrm{d}\overline{H}_i)/L_i \tag{14}$$

where $\mathrm{d}\overline{K}_{Ti}/\mathrm{d}\overline{H}_i = 1/\overline{H}_{0i}$.

Replacing the above derivative in Eq. (13) it follows

$$\delta \text{LCS} = P_1 C_{F1} F_R(\tau\alpha)_n(F'_R/F_R)A_c \times \sum_{i=1}^{12} g(Y_i) N_i \psi'(\overline{K}_{Ti})\delta \overline{H}_i. \tag{15}$$

According to Eq. (A.1) of Appendix A and Eq. (15), the uncertainty of LCS can be written as follows

$$\delta \text{LCS} = P_1 C_{F1} F_R(\tau\alpha)_n(F'_R/F_R)A_c \times \left[\sum_{i=1}^{12} g(Y_i)g(Y_j)\psi'(\overline{K}_{Ti})\psi'(\overline{K}_{Tj}) \times \rho_{ij} N_i N_j \delta \overline{H}_i \delta \overline{H}_j\right]^{1/2}. \tag{16}$$

Multiplying and dividing the sum in brackets of Eq. (16) by $N\overline{H}_a$, and dividing this equation by LCS it follows

$$\delta \text{LCS}/\text{LCS} = [P_1 C_{F1} F_R(\tau\alpha)_n(F'_R/F_R)A_c \overline{H}_a N/\text{LCS}] \times \left[\sum_{i=1}^{12} g(Y_i)g(Y_j)\psi'(\overline{K}_{Ti})\psi'(\overline{K}_{Tj}) \times \rho_{ij}(N_i/N)(N_j/N)(\delta \overline{H}_i/\overline{H}_a)(\delta \overline{H}_j/\overline{H}_a)\right]^{1/2} \tag{17}$$

where $\psi' = d\psi/d\overline{K}_T$ is given in Appendix B and $\overline{H}_a$ is the annual average of the long term monthly means $\overline{H}_i^*$. The correlation coefficient for the monthly means $\overline{H}_i$ and $\overline{H}_j$ can be defined according to Appendix A as follows

$$\rho_{ij} = \left[\sum_{k=1}^{M} (\overline{H}_{ik} - \overline{H}_i^*)(\overline{H}_{jk} - \overline{H}_j^*)/M \right] \Big/ \sigma_i \sigma_j \quad (18)$$

where

$$\sigma_i = \left[\sum_{k=1}^{M} (\overline{H}_{ik} - \overline{H}_i^*)^2 / M \right]^{1/2} \quad (19)$$

and

$$\overline{H}_i^* = \sum_{k=1}^{M} \overline{H}_{ik}/M \quad (20)$$

where M is the number of monthly averages (years) for months (i) and (j).

The computation of ρ_{ij} requires yearly series of qualified monthly means $\overline{H}_i$ with stabilized statistics, which means $M \geq 30$. The uncertainty analysis can be extended to M less than 30, if appropriate criteria are assumed to estimate the confidence interval (Coleman and Glenn Steele, 1989).

2.2. Case B: PV system integrated to the utility grid

The LCS in this case can be simplified by using the $P_1 - P_2$ method, once the average efficiency of the PV system η_i for each month (i) is known. In this case LCS is given by

$$\mathrm{LCS} = P_1 C_{E1} \sum_{i=1}^{12} \eta_i(\overline{T}_{pi}) \overline{H}_{Ti} N_i A_c - P_2(C_A A_c + C_E) \quad (21)$$

where C_{E1} is the electrical energy cost in the first year of the economical analysis (US\$/kWh) and $\overline{T}_{pi}$ is the average operating temperature of the PV modules for month (i). As previously mentioned, for amorphous silicon thin film PV temperature effects can be neglected.

Following the same steps that gave rise to Eq. (17), the uncertainty of LCS given by Eq. (21), can be written as follows

$$\delta\mathrm{LCS}/\mathrm{LCS} = (P_1 C_{E1} A_c \overline{H}_a N/\mathrm{LCS}) \times \left[\sum_{i,j=1}^{12} \eta_i \eta_j \psi'(\overline{K}_{Ti}) \psi'(\overline{K}_{Tj}) \rho_{ij} (N_i/N)(N_j/N) \times (\delta\overline{H}_i/\overline{H}_a)(\delta\overline{H}_j/\overline{H}_a) \right]^{1/2}. \quad (22)$$

Following Appendix A, if it is assumed a bias error B_i and an unbiased estimator S_i for σ_i, the precision index for a 95% confidence interval of $\overline{H}_i$ is tS_i, where t is the t-distribution of Student. By analogy with Eqs. (A.2) and (A.3), Eqs. (17) and (22) hold to estimate the bias error B_{LCS}, in which case $\delta\overline{H}_i$ should be replaced by B_i. These equations also hold to estimate the corresponding precision index of LCS, tS_{LCS}, in which case $\delta\overline{H}_i$ should be replaced by tS_i. The total uncertainty of LCS is then given by $U_{\mathrm{LCS}}^2 = B_{\mathrm{LCS}}^2 + (tS_{\mathrm{LCS}})^2$. The relative total uncertainty of LCS is defined here by $u_{\mathrm{LCS}} = U_{\mathrm{LCS}}/\mathrm{LCS}$, while the relative total uncertainty of $\overline{H}_i$ is defined by $u_{\overline{H}_i} = U_i/\overline{H}_a$, where $U_i^2 = B_i^2 + (tS_i)^2$. If the same relative uncertainty $u_{\overline{H}}$ is assumed for all monthly means $\overline{H}_i$, the square of $u_{\overline{H}}$ in Eqs. (17) and (22) can be brought out from the square root of the sum in the brackets, and therefore the mentioned equations can be cast in a single equation as follows

$$u_{\mathrm{LCS}} = P_E Q u_{\overline{H}} \quad (23)$$

where P_E and Q are given by

$$P_E = P_1 C_{F1} F_R(\tau\alpha)_n (F'_R/F_R) A_c \overline{H}_a N/\mathrm{LCS} \quad (24)$$

$$Q = \left[\sum_{i,j=1}^{12} g(Y_i) g(Y_j) \psi'(\overline{K}_{Ti}) \psi'(\overline{K}_{Tj}) \rho_{ij} (N_i/N) \times (N_j/N) \right]^{1/2} \quad (25)$$

for the case of SDHWS, and for the PV case,

$$P_E = P_1 C_{E1} A_c \overline{H}_a N/\mathrm{LCS} \quad (26)$$

$$Q = \left[\sum_{i,j=1}^{12} \eta_i \eta_j \psi'(\overline{K}_{Ti}) \psi'(\overline{K}_{Tj}) \rho_{ij} (N_i/N) \times (N_j/N) \right]^{1/2}. \quad (27)$$

As can be seen from Eq. (23), the factor P_E given by Eqs. (24) and (26) is a meaningful economical parameter. This factor is proportional to the ratio of the maximum energy saving due to solar radiation in the first year of the economical analysis, and the life cycle savings. The relative uncertainty of LCS is seen to be linear in $u_{\overline{H}}$. However it depends on the monthly means $\overline{H}_i$ through functions $g(Y_i)$, $\psi'(\overline{K}_{Ti})$, and f_i.

3. NUMERICAL EXAMPLES

In order to simplify the present analysis and to reduce the calculations, the relative uncertainty of all monthly means are assumed to be constant and equal to $u_{\overline{H}}$. Furthermore, the uncertainties of $\overline{H}_{Ti}$

as a function of $\overline{H}_i$ are not taken into account in the present analysis. The solar fraction f given by Eq. (3), as pointed out in Duffie and Beckman (1991), has an uncertainty around ±15%. This uncertainty will not be taken into account here either, so that the uncertainty estimates due to the monthly average radiation is only the first step to a comprehensive uncertainty analysis.

In the special case where the uncertainty of $\overline{H}_i$ is assumed to vanish for $12-p$ months, $p<12$, the calculations should be made for each case corresponding to the other p non-vanishing months. For $p=2$, there are $12!/10!2!=66$ cases; for $p=3$ there are $12!/9!3!=220$ cases, and so on. The total number of cases for all possible combinations of p non-vanishing months is the binomial number 2^{12}. With the assumption of the same relative uncertainty of $\overline{H}_i$ for all non vanishing p months, Eq. (23) still holds, in which case only the non-vanishing corresponding terms will be considered in the sum of Q. The slope of the resulting straight line of u_{LCS} as a function of $u_{\overline{H}}$, $P_E Q$, depends on the economical parameters, as the cost of the auxiliary energy as well as the way the monthly means are distributed during the year, and on the correlation between these monthly means. In particular, microclimate changes due to seasonal human activities, i.e. forest burning and also due to the activities of volcanoes can partially or totally impair the monthly means along the year. The impact of these activities on the uncertainty of LCS can also be estimated from Eq. (23).

The numerical examples are carried out here according to the following specifications.

Case A: SDHWS.

Collector area, A_c optimized for each location.

Annual load, $L=13.8$ GJ.

$F_R(\tau\alpha)_n = 0.7$.

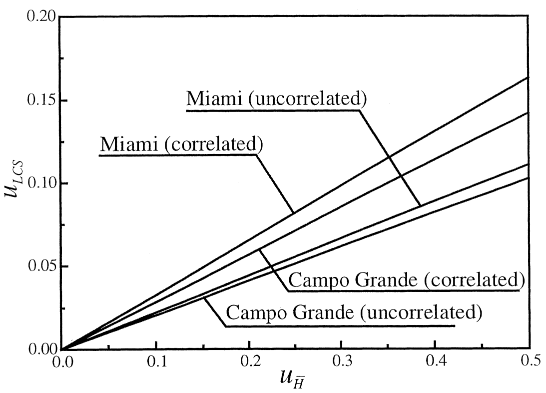

Fig. 1. Relative uncertainty of LCS, u_{LCS}, for the SDHWS for correlated and uncorrelated monthly means.

$F_R U_L = 5.0$ W/mK.

No heat exchanger ($F'_R = F_R$).

$\rho_g = 0.2$.

Cost of the fuel in the first year, $C_{F1}=$US\$ 28.00/GJ.

Inflation of C_{F1}, $i_F=10\%$.

Discount rate, $d=8\%$.

Cost of collector area, $C_A=$US\$ 85.00/m^2.

Cost independent of collector area, $C_E=$US\$ 600.00.

$P_1 = \mathrm{PWF}(N_e, i_F, d)$ (non-commercial plant).

$P_2 = 1$ (the system is totally financed by the owner).

Period of economical analysis, $N_e=20$ years.

In the present analysis, the cost C_E accounts for the cost of the storage tank. Customers may be interested in purchasing collectors in the circumstance they already have the tank (gas fueled or electrically heated). In this case, C_E would include only the installation cost, auxiliary pump and piping, and other minor costs.

Case B: PV system. The system analyzed here has been in operation since 1997 at LABSOLAR, and has the following specifications (Rüther, 1998, 1999):

Power = 2 kWp.

Average monthly efficiency, $\eta=5.3\%$ (measured).

$\rho_g = 0.2$.

Cost of the electrical energy, $C_{E1}=10$ ¢/kWh.

Inflation of C_{E1}, $i_F=10\%$.

Discount rate, $d=8\%$.

Capital cost = US\$ 14,000.00 (US\$ 7.00/Wp).

For the PV system chosen here, the threshold cost (for which LCS = 0) is US\$ 4.6/Wp.

We further detail our analysis for four locations. The first one is the location of Campo Grande (20.45° S, 54.62° W) in Brazil, and three other locations are in USA, namely, Miami (25.8° N, 80.27° W), Houston (29.98° N, 95.37° W) and Los Angeles (33.93° N, 118.4° W). Miami and Campo Grande are located in subtropical areas. For the location of Campo Grande, the Brazilian Weather Service (INMET) provided the records of monthly means of daily sums of global radiation, derived from measurements with pyranometers during the period between 1973 and 1990 (17 years). The monthly means derived from measured radiation for Miami, Houston and Los Angeles for a 30-year period is found in Marion and Wilcox (1994). The yearly long term monthly means $\overline{H}_i^*$, as well as the correlation coefficients for these locations are given in Appendix C. While the correlation coefficients ρ_{ij} are estimated with confidence for the USA locations, these

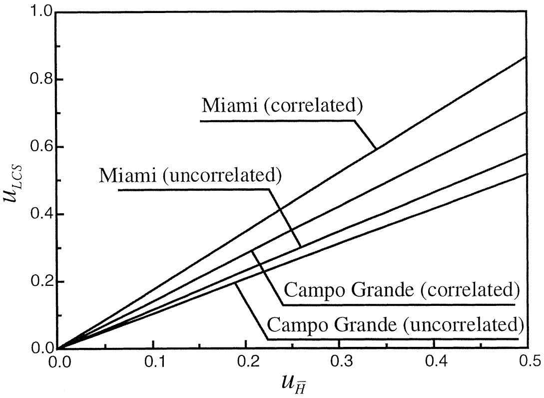

Fig. 2. Relative uncertainty of LCS for the PV system with capital cost of US\$ 3.00/Wp for correlated and uncorrelated monthly means.

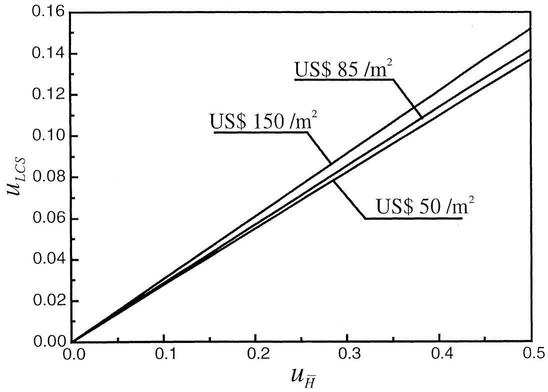

Fig. 4. Effect of the capital cost on the relative uncertainty of LCS for the SDHWS for the location of Campo Grande.

coefficients show a lesser degree of confidence for the location of Campo Grande, since the statistics for the 17-year time series is not stabilized. Therefore it is necessary to verify the effect of the correlation coefficients ρ_{ij} on the uncertainty of LCS. Figs. 1 and 2 show the results obtained from Eq. (23), for correlated ($\rho_{ij} \neq 0$) and uncorrelated ($\rho_{ij} = 1$; $i = j$ and $\rho_{ij} = 0$; $i \neq j$) monthly means, for the SDHWS and PV system, respectively, for Miami and Campo Grande. These figures show that for $u_{\bar{H}} = 10\%$ the uncertainty of the LCS differs in 2% for the SDHWS and around 8% for the PV system. This means that in the case of PV, when the capital cost is close to the threshold cost, the correlation coefficients should be significant in the evaluation of the uncertainty of LCS. The effect of the capital cost on the uncertainty of LCS is shown in Fig. 3, for the city of Los Angeles. It is seen from this figure that for a capital cost of US\$ 4.00/Wp, an uncertainty $u_{\bar{H}}$ of 10% corresponds to an uncertainty of LCS around

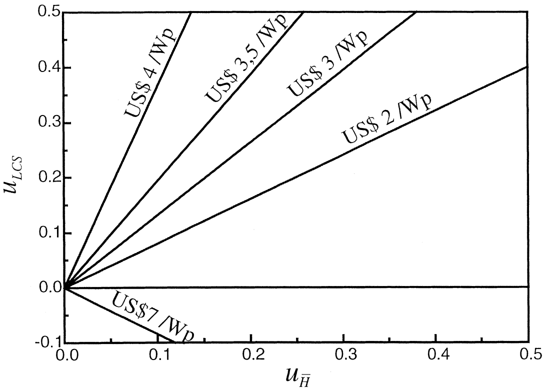

Fig. 3. Effect of the capital cost on the relative uncertainty of LCS for the PV system for the location of Los Angeles.

35%, while for a capital cost of US\$ 3.00/Wp it is around 10%. For a capital cost of US\$ 2.00/Wp, the uncertainty of LCS becomes pretty small, around 5% and for this case, $u_{\bar{H}}$ of 5% corresponds to an uncertainty of LCS around 3%.

Since the capital cost of the SDHWS is relatively low, the uncertainty $u_{\bar{H}}$ has a small effect on the uncertainty of LCS, as shown in Fig. 4. This is due to the relatively high value of LCS for the type of system chosen here.

The effect of the months for which the uncertainty vanishes is shown in Figs. 5–7, for the PV system with capital cost equal to US\$ 3.00/Wp. It can be seen from these figures that for p fixed non-vanishing uncertainties, all corresponding cases lie between two limiting straight lines, which correspond to the maximum and minimum for the set of all possible cases.

The uncertainty of LCS for the PV system for the different locations chosen is shown in Figs. 8 and 9. These figures show that for both the SDHWS and the PV system, the uncertainty of LCS depends on the location. This conclusion can be drawn for the USA locations chosen here. The results for Campo Grande are less precise, because of the lower confidence of the correlation coefficients of monthly means for this location.

The effect of electrical energy cost on the uncertainty of LCS is shown in Fig. 10 for the PV system with capital cost equal to US\$ 3.00/Wp. It shows how the increase in the electrical energy cost leads to a decrease in the uncertainty of LCS.

4. CONCLUSIONS

The uncertainty analysis of the LCS for a solar domestic hot water system and a PV system was

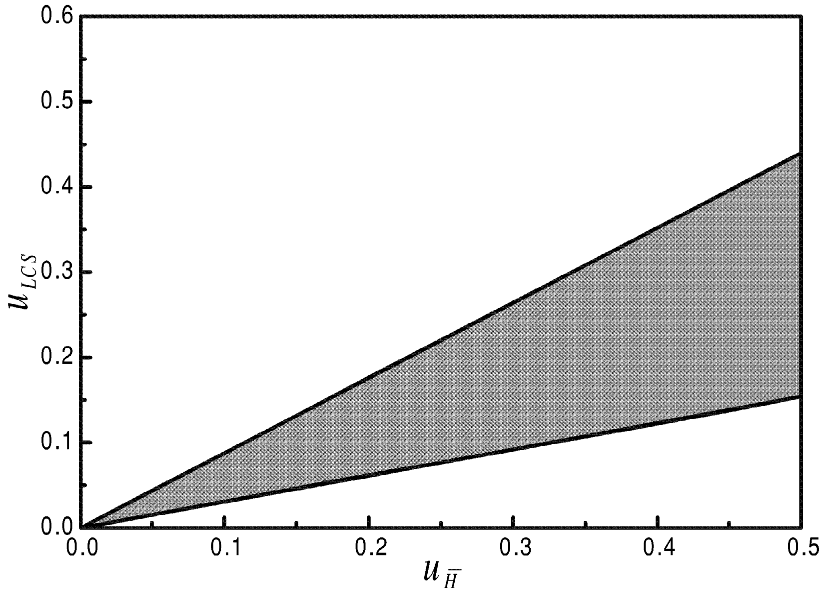

Fig. 5. Relative uncertainty scattering of LCS for the PV system, for capital cost of US$ 3.00/Wp, $p=3$ (220 cases), and the location of Los Angeles.

carried out. It is shown that the uncertainty of the monthly means of global radiation is important to estimate the uncertainty of the LCS of PV systems integrated to the utility grid, particularly in the case where the capital cost is close to the threshold cost. For a fixed value of the uncertainty of the LCS, there is a correlation between the uncertainty of the solar radiation data and the capital cost. The greater the capital cost, the smaller the accepted level of uncertainty of these data should be. The relative uncertainty of the LCS becomes sensitive with the uncertainty of the monthly means, but it is dependent on the value of LCS itself. However, the relative uncertainty of LCS is highly sensitive for cases of low LCS, i.e. for circumstances of low auxiliary energy cost or

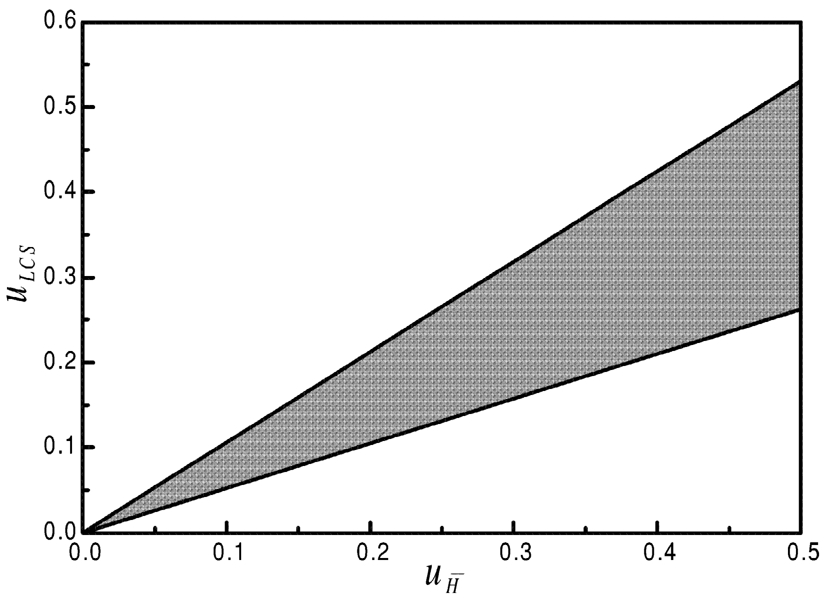

Fig. 6. Relative uncertainty scattering of LCS for the PV system, for capital cost of US$ 3.00/Wp, $p=6$ (924 cases), and the location of Los Angeles.

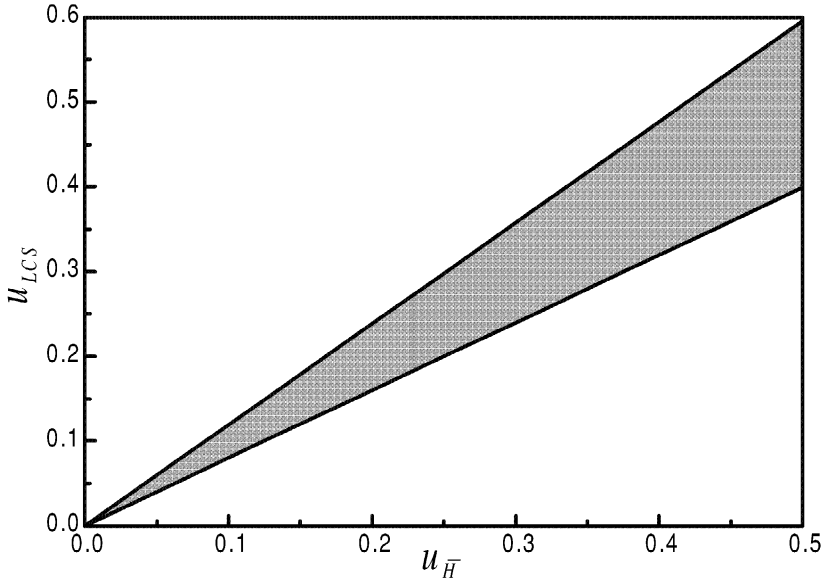

Fig. 7. Relative uncertainty scattering of LCS for the PV system, for capital cost of US$ 3.00/Wp, $p=9$ (220 cases), and the location of Los Angeles.

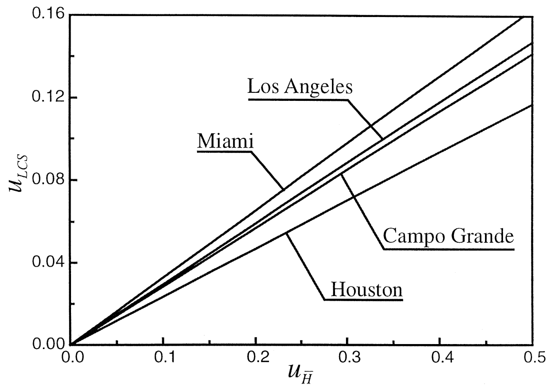

Fig. 8. Relative uncertainty of LCS for the SDHWS for different locations.

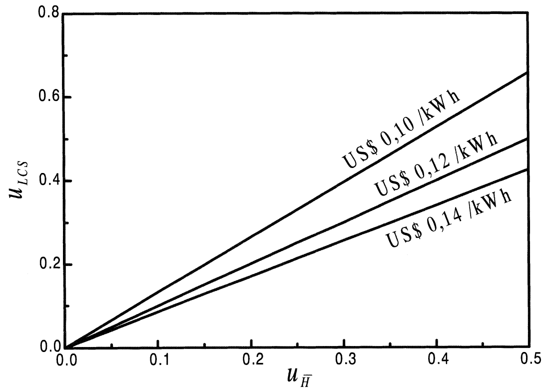

Fig. 10. Effect of the electrical energy cost on the relative uncertainty of LCS for the PV system, for capital cost of US$ 3.00/Wp, and the location of Los Angeles.

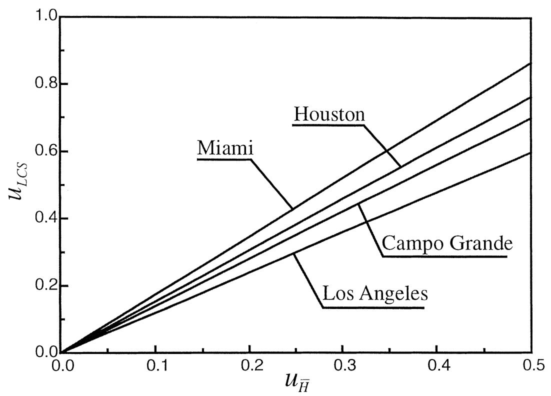

Fig. 9. Relative uncertainty of LCS for the PV system for capital cost of US$ 3.00/Wp, for different locations.

high capital cost. The effect of the uncertainty of a subset of months with known uncertainty in the monthly mean in the year is also shown for different subset cases.

The present analysis could be useful to determine the effect of the hourly variation of the electricity cost during the day, on the uncertainty of LCS for those cases of high effective load carrying capacity. The same approach used here can be extended, to analyze the uncertainty arising from the correlation relating the monthly mean of incident radiation on tilted surfaces to the monthly mean of global radiation on a horizontal surface. The uncertainty degree, levelly presented here is far underestimated, since the uncertainty associated with the correlation between global and diffuse radiation, as well as the uncertainty related to the f-chart are not taken into account in the present analysis either.

NOMENCLATURE

A_c	collector area (m^2)
B	bias error
C	$C = 0$ for non-commercial plants; $C = 1$ for commercial plants
C_{E1}	cost of electric energy in the first year of the period of economical analysis (US$/kWh)
C_{F1}	cost of the auxiliary energy in the first year of the period of economical analysis (US$/kWh)
d	discount rate
F_R	collector heat removal factor
F'_R	modified heat removal factor ($=F_R$)
$F_R U_L$	collector loss factor
$\bar{H}_a$	historical annual daily average of global radiation, derived from the monthly means $\bar{H}_i^*$ (J/m^2)
$\bar{H}_i$	monthly mean of global solar radiation incident on a horizontal surface for month (i) (J/m^2)
$\bar{H}_0$	monthly mean of solar radiation incident on a horizontal surface at the top of the atmosphere (J/m^2)
$\bar{H}_i^*$	historical average of the monthly means $\bar{H}_i$
$\bar{H}_{Ti}$	monthly mean of solar radiation incident on a tilted surface for month (i) (J/m^2)
i_F	inflation of the auxiliary energy
K_T	daily clearness index $= H/H_0$
$\bar{K}_T$	monthly average clearness index $= \bar{H}/\bar{H}_0$
L	annual load (GJ)
L_i	monthly load of month (i) (GJ)
LCS	life cycle savings
N	number of days in the year
N_i	number of days in the month (i)
N_e	period of the economical analysis
P_1	$P_1 = (1 - C\,\bar{t})\mathrm{PWF}(N_e, i_F, d)$
$P_2 = 1$	for the case the owner pays the system cash, non-commercial plant, no depreciation value, no federal and state taxes and no insurance cost
PWF	present worth factor for a series of payments
$\bar{R}_b$	ratio between the monthly mean of beam radiation incident on the tilted surface and the monthly mean of the beam radiation incident on the horizontal surface
tS	precision index
t	t-Student distribution
$\bar{t}$	effective income tax rate
$\bar{T}_a$	average monthly ambient temperature (°C)
$\bar{T}_{\mathrm{ref}}$	reference temperature for f-chart (100°C)
U	total uncertainty
u_{LCS}	relative uncertainty of $\mathrm{LCS} = U_{\mathrm{LCS}}/\mathrm{LCS}$
$u_{\bar{H}}$	relative uncertainty of $\bar{H} = U_{\bar{H}}/\bar{H}_a$

Greek symbols

Δt	total number of seconds in the month considered
ρ_g	reflectance of the ground surrounding the collectors
$\overline{(\tau\alpha)}$	average transmittance–absorptance product (monthly)
$(\tau\alpha)_b$	transmittance–absorptance product for beam radiation
$(\tau\alpha)_d$	transmittance–absorptance product for diffuse radiation
$(\tau\alpha)_g$	transmittance–absorptance product for radiation reflected from the ground
$(\tau\alpha)_n$	normal transmittance–absorptance product
ω_s	sunset angle for horizontal surfaces
β	tilt angle

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APPENDIX A. BASIC UNCERTAINTY ANALYSIS

Let $f = f(X_1, X_2, \ldots, X_n)$ be a function of n variables. Associated to each variable there is a bias error B_i and a variance σ_i. The total uncertainty for a 95% confidence interval associated to an estimate of X_i is $U_i^2 = B_i^2 + (tS_i)^2$, where S_i is an unbiased estimator for σ_i, tS_i is the precision index and t is the t-distribution of Student corresponding value, chosen for 95% confidence. Similarly, the uncertainty for f is defined as follows, $U_f^2 = B_f^2 + (tS_f)^2$. The relationship between U_f^2 and the uncertainties U_i^2, $i = 1, 2, \ldots, n$ according to Coleman and Glenn Steele (1989) is derived from the following differential form,

$$(\delta f)^2 = \sum_{i,j=1}^{n} (\partial f/\partial X_i)(\partial f/\partial X_j)\rho_{ij}\delta X_i \delta X_j \tag{A.1}$$

from which the following equations are obtained to estimate B_f and tS_f,

$$B_f^2 = \sum_{i,j=1}^{n} \partial f/\partial X_i \partial f/\partial X_j \rho_{ij} B_i B_j \tag{A.2}$$

$$(tS_f)^2 = \sum_{i,j=1}^{n} \partial f/\partial X_i \partial f/\partial X_j \rho_{ij}(tS_i)(tS_j) \tag{A.3}$$

where

$$\rho_{ij} = \sigma_{ij}/\sigma_i \sigma_j \tag{A.4}$$

σ_{ij} is the covariance of X_i and X_j given by

$$\sigma_{ij} = \lim_{N\to\infty} \sum_{k=1}^{N} (X_{ik} - \mu_i)(X_{jk} - \mu_i)/N \tag{A.5}$$

where $\sigma_{ij} = \sigma_{ji}$ and

$$\mu_i = \lim_{N\to\infty} \sum_{k=1}^{N} X_{ik}/N \tag{A.6}$$

is the expected value of X_i.

If the distribution of $\varepsilon_{ik} = X_{ik} - \mu_i$ is normal, for $N > 30$ the estimator of μ_i is given by

$$\bar{X}_i = \sum_{k=1}^{N} X_{ik}/N \tag{A.7}$$

while σ_{ij} is estimated by

$$\sigma_{ij} = \sum_{k=1}^{N} (X_{ik} - \bar{X}_i)(X_{jk} - \bar{X}_i)/N \tag{A.8}$$

and σ_i is given by

$$\sigma_i^2 = \sum_{k=1}^{N} (X_{ik} - \bar{X}_i)^2/N. \tag{A.9}$$

In the case the same bias error B and precision index tS are assumed for all X_i, the sum of Eqs. (A.2) and (A.3) leads to

$$U_f = \left(\sum_{i,j=1}^{n} \partial f/\partial X_i \partial f/\partial X_j \rho_{ij} \right)^{1/2} U \tag{A.10}$$

where $U^2 = B^2 + (tS)^2$.

For uncorrelated variables X_i and X_j, i.e. $\rho_{ij} = 0$ for $i \neq j$, the sum of Eqs. (A.2) and (A.3) leads to

$$U_f = \left[\sum_{i,j=1}^{n} (\partial f/\partial X_i)^2 U_i^2 \right]^{1/2} \tag{A.11}$$

APPENDIX B. CORRELATION FOR $\bar{H}_T$

The correlation of Hay (1979), between $\bar{H}_T$ and $\bar{H}$ can be expressed in the dimensionless form as

$$\begin{aligned}\psi(\bar{K}_T) &= (\overline{\tau\alpha})/(\tau\alpha)_n \bar{H}_T/\bar{H}_0 \\ &= (\bar{K}_T - \varphi(\bar{K}_T))(\overline{\tau\alpha})_b/(\tau\alpha)_n \bar{R}_b \\ &\quad + \rho_g (\overline{\tau\alpha})_g/(\tau\alpha)_n \bar{K}_T (1 - \cos\beta)/2 \\ &\quad + (\overline{\tau\alpha})_d/(\tau\alpha)_n \varphi(\bar{K}_T) \\ &\quad \times \{(\bar{K}_T - \varphi(\bar{K}_T))\bar{R}_b + (1 + \cos\beta)/2[1 \\ &\quad - (\bar{K}_T - \varphi(\bar{K}_T))]\}\end{aligned} \tag{B.1}$$

where $\varphi = \bar{H}_d/\bar{H}_0$ is expressed according to Erbs *et al.* (1982) as follows

$$\varphi(\bar{K}_T) = (\bar{H}_d/\bar{H})(\bar{H}/\bar{H}_0) = \begin{cases} 1.391\bar{K}_T - 3.560\bar{K}_T^2 + 4.189\bar{K}_T^3 - 2.137\bar{K}_T^4 \\ 0.3 \leq \bar{K}_T \leq 0.8, \text{ for } \omega_s \leq 81.4° \\ 1.311\bar{K}_T - 3.022\bar{K}_T^2 + 3.427\bar{K}_T^3 - 1.821\bar{K}_T^4 \\ 0.3 \leq \bar{K}_T \leq 0.8, \text{ for } \omega_s > 81.4° \end{cases} \tag{B.2}$$

where $\bar{K}_T = \bar{H}_T/\bar{H}_0$. The derivative of ψ is given by

$$\begin{aligned}\psi'(\bar{K}_T) &= [1 - \varphi'(\bar{K}_T)](\overline{\tau\alpha})_b/(\tau\alpha)_n \bar{R}_b \\ &+ \rho_g (\overline{\tau\alpha})_g/(\tau\alpha)_n (1 - \cos\beta)/2 + (\overline{\tau\alpha})_d/(\tau\alpha)_n \\ &\times \varphi'(\bar{K}_T)\{[\bar{K}_T - \varphi(\bar{K}_T)]\bar{R}_b + (1 + \cos\beta)/2 \\ &\times [1 - (\bar{K}_T - \varphi(\bar{K}_T))]\} + (\overline{\tau\alpha})_d/(\tau\alpha)_n \varphi(\bar{K}_T) \\ &\times \{[1 - \varphi'(\bar{K}_T)]\bar{R}_b + (1 + \cos\beta)/2[\varphi'(\bar{K}_T) - 1]\}\end{aligned} \tag{B.3}$$

where $\varphi'(\bar{K}_T) = d\varphi/d\bar{K}_T$.

APPENDIX C

See Tables C.1–C.5.

Table C.1. Correlation coefficients ρ_{ij} $(= \rho_{ji})$ for the location of Campo Grande, Brazil

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Jan	1	0.041	−0.602	0.349	−0.269	0.048	0.259	−0.400	0.318	−0.234	−0.806	−0.529
Feb		1	0.479	0.240	0.294	0.122	−0.296	−0.130	0.326	0.570	−0.147	0.009
Mar			1	0.226	−0.111	0.251	−0.384	−0.097	−0.234	0.517	0.217	−0.102
Apr				1	−0.007	−0.275	−0.594	−0.690	−0.260	0.044	−0.336	−0.416
May					1	−0.104	−0.032	−0.119	0.533	0.489	0.272	0.216
Jun						1	0.620	−0.070	0.644	0.547	0.272	0.227
Jul							1	0.161	0.569	0.001	0.181	−0.448
Aug								1	0.123	−0.081	0.755	0.689
Sept									1	0.592	0.386	0.513
Oct										1	0.389	0.310
Nov											1	0.635
Dec												1

Table C.2. Correlation coefficients ρ_{ij} $(=\rho_{ji})$ for the location of Houston, US

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Jan	1	0.180	0.068	−0.004	−0.081	−0.022	−0.110	0.123	0.123	−0.363	−0.191	−0.266
Feb		1	−0.048	0.160	0.174	0.158	0.186	−0.185	−0.484	−0.387	−0.307	−0.329
Mar			1	0.359	−0.044	−0.142	−0.002	−0.225	−0.285	0.191	−0.176	−0.342
Apr				1	0.232	0.110	0.158	0.068	−0.112	0.016	−0.038	−0.045
May					1	0.006	0.056	0.258	−0.139	−0.160	0.131	0.285
Jun						1	0.019	0.052	0.051	0.279	0.149	0.024
Jul							1	0.098	−0.182	−0.248	−0.087	0.008
Aug								1	0.230	−0.055	0.233	−0.008
Sept									1	0.106	0.327	0.259
Oct										1	0.389	0.123
Nov											1	0.374
Dec												1

Table C.3. Correlation coefficients ρ_{ij} $(=\rho_{ji})$ for the location of Los Angeles, US

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Jan	1	0.197	0.238	−0.247	0.206	−0.103	−0.400	0.077	−0.092	0.123	0.042	−0.047
Feb		1	0.166	0.044	0.180	0.239	0.035	0.040	0.171	0.061	−0.120	−0.297
Mar			1	0.172	0.132	0.009	−0.327	0.065	0.061	−0.037	−0.080	−0.254
Apr				1	−0.135	−0.027	−0.132	−0.031	0.087	−0.472	0.120	−0.047
May					1	0.166	0.060	0.023	0.381	−0.041	0.095	−0.119
Jun						1	0.024	0.259	0.014	0.000	0.240	0.258
Jul							1	0.139	0.263	0.132	−0.087	−0.026
Aug								1	−0.093	0.100	−0.078	0.127
Sept									1	−0.177	0.152	−0.126
Oct										1	0.008	−0.002
Nov											1	0.382
Dec												1

Table C.4. Correlation coefficients ρ_{ij} $(=\rho_{ji})$ for the location of Miami, US

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Jan	1	0.190	0.292	0.146	0.265	0.290	0.085	−0.234	−0.080	0.103	−0.178	0.043
Feb		1	0.228	0.005	0.221	−0.058	0.334	−0.061	0.423	0.147	−0.065	0.264
Mar			1	0.494	0.122	−0.025	0.226	−0.047	0.015	0.439	−0.072	0.083
Apr				1	0.182	0.223	0.401	0.297	0.358	0.442	−0.090	−0.046
May					1	0.250	0.290	−0.082	0.274	0.066	−0.210	0.109
Jun						1	0.391	0.020	0.140	0.163	−0.284	−0.060
Jul							1	0.302	0.266	0.112	−0.160	0.133
Aug								1	0.285	−0.196	0.089	0.228
Sept									1	0.358	0.039	0.218
Oct										1	−0.054	0.087
Nov											1	0.631
Dec												1

Table C.5. Yearly average of monthly means of daily solar radiation, $\overline{H}_i^*$ (MJ/m^2)

	Campo Grande	Houston	Los Angeles	Miami
Jan	20.9	9.7	10.5	12.6
Feb	20.0	12.2	13.8	15.1
Mar	19.2	15.5	18.4	18.7
Apr	17.8	18.0	22.2	21.6
May	14.7	20.2	23.4	21.6
Jun	13.8	21.6	24.1	20.2
Jul	15.4	21.6	26.2	20.9
Aug	16.2	20.2	23.6	20.2
Sept	17.5	17.6	19.1	17.6
Oct	20.8	15.1	15.0	15.8
Nov	22.0	11.2	11.4	13.3
Dec	20.8	9.0	9.6	11.9

REFERENCES

Behr H. D. (1997) Solar radiation on tilted south oriented surfaces: validation of transfer-models. *Solar Energy* **61**(6), 399.

Brandemuehl M. J. and Beckman W. A. (1979) Economic evaluation and optimization of solar heating systems. *Solar Energy* **23**(1), 1.

Bruton T. M., Woodcock J. M., Roy K., Garrard P., Alonso J., Nijs J., Räuber A., Vallera A., Schade H., Alsema E., Hill R. and Dimmler B. (1996) Multi-megawatt upscaling of silicon and thin film solar cell and module manufacturing 'MUSIC FM'. In *APAS RENA Report CT94 0008*, p. 1.

Coleman H. W. and Glenn Steele Jr. W. (1989). *Experimentation and Uncertainty Analysis for Engineers*, Wiley, Interscience, New York.

Colle S., Abreu S. L. and Rüther R. (1999a) Uncertainty in economical analysis of solar water heating and photovoltaic systems. In *Proceedings of the ISES Solar World Congress, Jerusalem, Israel*, In press.

Colle S., Abreu S. L., Couto P., Mantelli S., Pereira E. B., Raschke E. and Stuhlmann R. (1999b) Distribution of solar irradiation in Brazil derived from geostationary satellite data. In *Proceedings of the ISES Solar World Congress, Jerusalem, Israel*, In press.

Duffie J. A. and Beckman W. A. (1991). In 2nd edn, Solar Engineering of Thermal Processes, Wiley Interscience, New York.

Erbs D. C., Klein S. A. and Duffie J. A. (1982) Estimation of the diffuse radiation fraction for hourly daily and monthly-averages global radiation. *Solar Energy* **28**, 293.

Hay J. E. (1979) Calculation of monthly mean solar radiation for horizontal and tilted surfaces. *Solar Energy* **23**, 301.

Hay J. E. and McKay D. C. (1985) Estimating solar irradiance on tilted surfaces: a view and assessment of methodologies. *Int. J. Solar Energy* **3**, 203.

Marion W. and Wilcox S. (1994). *Solar Radiation Data Manual for Flat-plate and Concentrations Collectors*, NREL–US Department of Energy, 463-5607 DE93018229.

Pereira E. B., Abreu S. L., Stuhlmann R., Rieland M. and Colle S. (1996) Survey of the incident solar radiation in Brazil by use of meteosat satellite data. *Solar Energy* **57**(2), 125.

Pinker R. T. and Lazlo I. (1992) Modelling surface solar irradiance for satellite applications on a global scale. *J. Appl. Meteorol.* **32**, 194.

Reindl D. T., Beckman W. A. and Duffie I. A. (1990) Evaluation of hourly tilted surface radiation models. *Solar Energy* **45**(1), 9.

Rüther R. (1996) Advantages of thin film solar modules in façade, sound barrier and roof-mounted PV systems. In *Proceedings of EuroSun'96, Freiburg, Germany*, p. 862.

Rüther R. (1998) Experiences and operational results of the first grid-connected, building-integrated, thin film photovoltaic installation in Brazil. In *Proceedings of the 2nd World Conference on Photovoltaic Solar Energy Conversion, Vienna, Austria*, p. 2655.

Rüther R. (1999) Demonstrating the superior performance of thin film amorphous silicon for building-integrated photovoltaic systems in warm climates. In *Proceedings of the ISES Solar World Congress, Jerusalem, Israel*, In press.

Rüther R. and Dacoregio M. M. (2000) Performance assessment of a 2 kWp grid-connected, building-integrated, amorphous silicon photovoltaic installation in Brazil. *Prog. Photovoltaics: Res. Appl.* **8**, 257.

Rüther R. (2000) The first grid-connected, building-integrated, thin film photovoltaic installation in Brazil: output performance after two years. In *Proceedings of the 16th European Photovoltaic Solar Energy Conference, Glasgow, Scotland*, In press.

Staebler D. L. and Wronski C. R. (1977) Reversible conductivity changes in discharge-produced amorphous Si. *Appl. Phys. Lett.* **31**, 292.

Stuhlmann R., Rieland M. and Raschke E. (1990) An improvement of the IGMK Model to derive total and diffuse solar radiation at the surface from satellite data. *J. Appl. Meteorol.* **18**, 1172.

Tsvetkov, A. (1997). Personal communication at the BSRN Workshop of Budapest, May 1997. Reports of the World Radiation Data Centre, Leningrad (St. Petersburg).

Zelenka A., Czeplak G., D'Agostino V., Josefsson W., Maxwell E., Perez R., Noia M., Ratto C. and Festa R. (1992) Techniques for supplementing solar radiation network data. In *Report IEA, SHCP 90-1, 2*, International Energy Agency.

Zweibel K. (1999) Issues in thin film PV manufacturing cost reduction. *Solar Energy Mater. Solar Cells* **59**, 1.